Study of the Effect of Crack Location in Bi-Material Plates

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Abstract

The Bi-Material structures are characterized by the fact that they possess a specific behavior. This feature often requires complex formulations to achieve realistic mechanical behavior. In addition, if there is a crack in the interface and if it reaches a critical condition, the crack may extend along the interface, provided that the interface is sufficiently weak compared to one or the other material. The mechanical modeling of interfacial cracks is carried out by the extended finite element method (X-FEM).

The objective of this paper is to study the location of crack in bi-material plates. We took several locations of cracks. The results obtained are satisfactory.

Keywords: Cracks, Interface, Bi-Material, X-FEM, Locations

Introduction

The fracture mechanics of interface cracks in multi-material systems has received considerable attention. The mixed mode nature of the stress and displacement field at the crack tip, the oscillatory nature of the singularity, and the inseparability of strain energy release rate modes has been well documented in Hutchinson and Suo (1992) [1] and Rice (1988) [2]. Hutchinson and Suo[1] also discuss a methodology for characterizing the fracture toughness of an interface in plane, bi-material problems.

A number of methods have been advanced for the numerical determination of interface stress intensity factors in plane bi-material problems such as finite element method (FEM) and boundary element method (BEM). Matos et al.[3] used the virtual crack extension method in conjunction with the FEM to solve bi-material fracture problem. Miyazaki et al.[4] applied the virtual crack extension method with the BEM. Lee and Choi [5] used the FEM to solve the problems of bi-materials cracks under different loading conditions. EFGM and XFEM have been used for the analysis of interface cracks between two different materials.[6]

In this context, this work aims to model the behavior of structures containing interfaces cracks and see their effect for different emplacement in the structure. The obtained results will be evaluated using two approaches; the first is global, based on the J integral and the other is local, based on the displacement jump at the crack edges near the crack tip.

Modeling of interface crack

The singular near-tip field for the interface crack (Figure 1) is given by[1, 7] as:

\[ \sigma_{22}^\infty + r_{12}^\infty = \frac{K}{\sqrt{2\pi r}} r^{ic} \]

where \( K = K_1 + iK_2 \) is the complex interface stress intensity factor, \( r \) is the distance from the crack tip, and:

\[ \varepsilon = \frac{1}{2\pi} \log \left( 1 + \beta \right) \]

which is a function of the second Dundurs parameter \( \beta \)[8]:

\[ \beta = \frac{\mu_1 (k_2 - 1) - \mu_2 (k_1 - 1)}{\mu_1 (k_2 + 1) + \mu_2 (k_1 + 1)} \]

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And the first the second Dundurs parameter $\alpha$ [8] is given by:

$$\alpha = \frac{\mu_1 (k_2 + 1) - \mu_2 (k_1 + 1)}{\mu_1 (k_2 + 1) + \mu_2 (k_1 + 1)}$$

(4)

The phase angle is a measure of the relative proportion of shear to normal tractions ahead of the crack tip. It is defined through the relation [9]:

$$\omega = \tan^{-1} \left( \frac{K_2}{K_1} \right)$$

(5)

The phase angle is an important parameter in the characterization of interfacial fracture toughness. In reporting the phase angle for a given loading configuration, two techniques are used in this work; the J integral method and the displacement jump method [10, 11].

For the first method, we use the interaction integral defined as [6]:

$$I = \int \left( \sigma_{ik} \delta_{ij} u_i - \sigma_{ij} u_i \right) \nu_{jk} d\Gamma, \quad k=x \text{ or } y$$

(8)

$$I = \frac{2}{E^*} K_i (1) K_j (2) + K_i (2) K_j (1)$$

(9)

where:

$$E^* = \frac{E_1}{1 - \nu_1^2} \quad \text{Plane stress}$$

$$E^* = \frac{E_1}{1 - \nu_2^*} \quad \text{(i = 1, 2)}$$

(10)

Then the stress intensity factor for interfaces cracks becomes [6]:

$$K_i = \frac{E^* \cos^2(\pi x)}{2} I_1, \quad K_2 = \frac{E^* \cos^2(\pi x)}{2} I_2$$

(11)

where: $I_{\text{model}}$ is the interaction integral of mode $I$.

The second method is based on the displacement jump between the two crack sides by employing the displacement field near the crack tip. The mixed SIF is deduced by Nehar et al. [12] as:

$$\left\{ \begin{array}{c} K_1 = D \left[ \begin{array}{c} 2 \cos(\text{ln}(x)) - \sin(\text{ln}(y)) \end{array} \right] \left[ \begin{array}{c} \Delta u_\nu \\ -\cos(\text{ln}(x)) + 2 \sin(\text{ln}(y)) \end{array} \right] \\
K_2 = D \left[ \begin{array}{c} -\cos(\text{ln}(x)) + 2 \sin(\text{ln}(y)) \\ -\cos(\text{ln}(x)) - \sin(\text{ln}(y)) \end{array} \right] \left[ \begin{array}{c} \Delta u_\nu \\ \Delta u_\nu \end{array} \right] \end{array} \right.$$

(12)

with:

$$D = -\frac{2 \mu_1 \mu_2 \cos(\pi x)}{\mu_1 (1 + k_1) + \mu_2 (1 + k_2) E_2} \sqrt{\frac{1}{r}}$$

(13)

where: $\Delta u_\nu$, $\Delta u_\nu$ are the displacement jump.

$$k_j = \begin{cases} \frac{3 - V_1}{4 + V_1} & (\text{For plane stress}) \\ \frac{3 - V_1}{1 + V_1} & (\text{For plane strain}) \end{cases} \quad (i = 1, 2)$$

(14)

$$\mu_i = \frac{E_i}{2(1 + V_i)} \quad (i = 1, 2)$$

(15)

### Numerical example

Consider an infinite bi-material plate between two rigid grips ($H=4$m) with an edge crack of a length of $2$m. In the first case we take the crack driven by the relative translation of the two grips and in the second case we take a sub-interface crack, $h=1$m (Fig 1.a et b). The grips are assumed to be perfectly rigid so that no separation no sliding takes place between the bi-material plate and the grips.

The material properties are given by:

- Modulus of elasticity: $E_1 = 22 E_2$
- Poisson’s ratio: $\nu_1 = 0.26$, and $\nu_2 = 0.30$
A parametric study of the crack emplacement in the plate is conducted. A calculation of the phase angle of the bi-material plate is performed.

Results and discussions

A bi-material interface crack between rigid grips

In Table 1, the results of the phase angle are listed. They are computed using the J integral and the displacement jump methods then they are compared to reference solution of Hutchinson et al. [1]. The Dunturs parameter $\beta = \alpha/4$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Hutchinson and Suo [1]</th>
<th>X-FEM (J Integral)</th>
<th>X-FEM (Disp. Jump)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-3.26</td>
<td>-3.33</td>
<td>-3.38</td>
</tr>
<tr>
<td>0.4</td>
<td>-6.42</td>
<td>-6.46</td>
<td>-6.51</td>
</tr>
<tr>
<td>0.6</td>
<td>-10.20</td>
<td>-9.66</td>
<td>-9.59</td>
</tr>
<tr>
<td>0.8</td>
<td>-13.6</td>
<td>-12.96</td>
<td>-12.5</td>
</tr>
</tbody>
</table>

We have presented the results of the table 1 in the Figure 2.

A sub interface crack

And for the second case of crack emplacement (sub-interface crack), the obtained results of phase angle calculated using the J integral, the displacement jump methods and the reference results of Hutchinson [1] are regrouped in the table 2. In this case the Dunturs parameters $\beta$ is taken also $\alpha/4$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Hutchinson and Suo [1]</th>
<th>X-FEM (J Integral)</th>
<th>X-FEM (Disp. Jump)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1.3902</td>
<td>1.46</td>
<td>1.39</td>
</tr>
<tr>
<td>0.4</td>
<td>3.1829</td>
<td>2.79</td>
<td>2.64</td>
</tr>
<tr>
<td>0.6</td>
<td>4.8902</td>
<td>4.06</td>
<td>3.83</td>
</tr>
<tr>
<td>0.8</td>
<td>5.8293</td>
<td>5.31</td>
<td>5</td>
</tr>
</tbody>
</table>

We have presented also the results of the table 2 in the Figure 3.
In addition, the location of the crack influences on the results of the phase angle which is very clear in the figures 3 and 3 such that for a crack in the interface the results of the phase angle decrease linearly with the increase of the Dunturs parameter $\alpha$ while for a crack is under the interface the phase angle in this case increases which means that the existence of a crack in the interface is more dangerous than the second case.

**Conclusion**

In this paper, we have proposed the study of bi-material interface cracks with the X-FEM method by applying two variants of approach: The J integral and the displacement jump. This work has oriented to the analysis of cracks emplacement in the plate that lie at or displaced of the interface of two elastically homogeneous isotropic materials.

The correlation of the obtained results with the literature for several treated configurations demonstrates the effectiveness of this procedure.

We can obtain these conclusions:

- The location of the crack influences on the results of the phase angle
- The results of the phase angle for crack in the interface decreases linearly with the increase of the Dunturs parameter $\alpha$
- The results of the phase angle for sub-interface crack increase
- The existence of a crack in the interface is more dangerous

**References**


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