Limit analysis of frames - Benchmarking and Applications

M Sailouhi, L Boushine, KI Janati, AM Dirhar

Abstract

The analysis limit is currently experiencing developments and integrates increasingly into building codes. In this article, the methods of analysis limit of frames are exposed, particularly the equilibrium and the work methods. Then an application to a 2D frame is performed by the finite element program Plimit [1], the results obtained, i.e. the load limit that the structure can carry and the sequence of formation of plastic hinges, are discussed and compared to those obtained by commercial software (SAP 2000). The influence of the addition of the cross of St. Andrew on the limit load was studied. Finally, an example of a 3D frame is exposed.

Introduction

Steel frames show a high nonlinear behavior due to the plasticity of the material and the slenderness of members. How to approach the plastic behavior of steel frames has been a large subject in the research field of constructional computation, and specially, the load carrying capacity.

The theory of limit analysis aims to determine, for a given structure and loading, load factor producing ruin.

A special case of the analysis limit for buildings is the static method ‘pushover’ [3]. This is a simple option to estimate the resistance capacity in the post-elastic range. [4] The technique can also be used to highlight potential weak points in the structure. The method includes applying a predetermined lateral load which is distributed along the building height. The lateral forces are then increased monotonically until a level of displacement, called target displacement [5].

Theory of limit analysis of plans frames

In the plastic analysis and design, two basic methods are commonly used: the “equilibrium method”, which is based on the theorem of the lower bound and the work method which is based, in turn, on the theorem of the upper terminal [6].

Equilibrium method

In the equilibrium method, the relationship between the strength of a structure and the applied loads is found by adjusting the unknown redundants in an indeterminate structure such that:

- The equilibrium condition is always satisfied
- The moment condition is not violated

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• The mechanism condition may or may not be satisfied

The equation formed is called the statically admissible “equilibrium equation”. It gives the relationship between the structure strength and the applied loads for a particular set of assumed redundant moments. Although any set of assumed redundants will give a safe or lower-bound solution.

However, the best set of redundant moments is the one that gives the largest applied load-carrying capacity (or the smallest required plastic moment) of the structure. This set of assumed redundant moments corresponds to the formation of a plastic failure mechanism.

For indeterminate structures, equilibrium equations always lead to the moment diagram in terms of unknown redundants. In the plastic analysis, the analyst can choose the values of unknown redundants in the moment equilibrium equations, which often lead to a quick safe solution to the problem. The solution will of course be exact only if the chosen values of the redundant moments result in a plastic collapse mechanism.

Work method

The relation between the strength of a frame and the applied loads in the work method is found by assuming that there is no overall loss of energy as the frame under failure loads undergoes a small change in displacement.

Thus, by postulating a valid failure mechanism, an equation can be formed by equating the external work done by the applied loads through the displacements to the internal dissipation of energy at the plastic hinge locations. The internal dissipation of energy is the sum of the products of the plastic moment at each hinge and the corresponding angular change required to effect a small movement of the failure mechanism.

The external work is the sum of the products of the component of the small displacement of the failure mechanism in line with the applied load and the corresponding applied load (Eq.1). The equation formed in this way is called the work equation and the corresponding collapse load or the required plastic moment capacity can be determined by solving the work equation 1 [7].

\[
\sum W \delta_i = \sum M_p \theta_i
\]  
(Eq. 1)

Where the left-hand summation extends over all the loads and the right-hand summation extends over all the plastic hinges.

The computed load for the particular assumed failure mechanism is exact if a moment check is performed and shows that the plastic moment condition is not violated anywhere in the frame.

For a given structure and an applied load, many collapses are possible. Every possible collapse mechanism can be obtained as some combination of a certain number of independent mechanisms.

If the number of independent mechanisms is known in advance, then the combination could be made in a systematic manner and there would be less likelihood of overlooking a possible combination. The number of possible independent mechanisms n for a frame can be determined from (Eq. 2).

\[
n = N - R
\]  
(Eq. 2)

Where N is the number of critical sections at which plastic hinges might form under the particular loading system and R is the degree of redundancy of the structure.

Once these independent mechanisms have been identified, a work equation is written for each combination and the corresponding collapse load is determined. The lowest load among those obtained by considering all the possible combinations of the independent mechanisms is the correct plastic limit or collapse load. The final confirmation of the validity of the best combination is made by performing a moment check, which may also indicate further adjustments that need to be made.

Program Plimit

The program Plimit concerns a non-linear analysis in which the moment curvature behavior of the members is assumed to be elastic-perfectly plastic as shown in Figure 1.

To deal with this non-linearity, an iterative approach is used to find the nodal displacements and element “actions” under a given set of applied loads. Moments in excess of their plastic limits are re-distributed to other joints which still have reserves of moment carrying capacity. Convergence of the iterative process is said to have occurred when, within certain tolerances, moments at the element nodes nowhere exceed their limiting plastic values, and the internal “actions” are in equilibrium with the applied
external loads.

The conventional approach for tackling this type of problem is progressively to modify the global stiffness matrix as joints reach the plastic limit. The modification is necessary because a plastic joint is replaced by a pin joint with the appropriate plastic moment applied. The global stiffness matrix is formed once only, with the non-linearity introduced by iteratively modifying the applied forces on the structure until convergence is achieved.

**Plimit and the sap 2000 software**

**Presentation of the structure**

The structure is a two-story rectangular (Fig.2). The height of each story is 10 m and the bay width is 20 m. All columns and beams have the same cross-section (IPE 600). The material is assumed elastic perfectly plastic with a yield stress of \(3,8 \times 10^8\) N/m\(^2\). So, the full plastic moment is about 1331.8 KN.m. The moment-curvature curve of IPE 600 is shown in Fig. 3. Other characteristics are listed in table 1.
The system of loading of the structure is composed of horizontal forces applied to the B and C (Fig. 2). The values of these forces are respectively 324 KN and 216 KN. It is proposed to determine the load factor $\lambda_{lim}$ producing the collapse of the structure.

Results obtained by the program Plimit

A limit analysis was performed using the Plimit program. The structure was subjected to a progressive load, through the increase in the loads applied by a load factor. For each load factor, the program determines the vector forces of each finite element (two efforts and one moment) and the corresponding displacement.

As loads on the structure rise, plastic moments are formed and “collapse” occurs when a mechanism is developed. The first hinges was appeared at the points A and H (Fig. 2) for a load factor of $\lambda_{el} = 0.75$, which corresponds to the elastic limit. The program allows, in fact, determine the location of plastic hinges but also the sequence of their formation (Tab. 2).

<table>
<thead>
<tr>
<th>Load factor</th>
<th>Resultant base shear KN</th>
<th>Number of plastic hinges</th>
<th>Location of plastic hinges</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>0.31</td>
<td>165.9</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>0.61</td>
<td>331.7</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>0.75</td>
<td>402.9</td>
<td>2</td>
<td>A H</td>
</tr>
<tr>
<td>0.85</td>
<td>460.2</td>
<td>2</td>
<td>A H</td>
</tr>
<tr>
<td>0.89</td>
<td>479.0</td>
<td>4</td>
<td>A H B3 G3</td>
</tr>
<tr>
<td>0.92</td>
<td>496.4</td>
<td>4</td>
<td>A H B3 G3</td>
</tr>
<tr>
<td>0.95</td>
<td>513.8</td>
<td>4</td>
<td>A H B3 G3</td>
</tr>
<tr>
<td>0.98</td>
<td>531.1</td>
<td>4</td>
<td>A H B3 G3</td>
</tr>
<tr>
<td>0.99</td>
<td>534.2</td>
<td>6</td>
<td>A H B3 G3 B1 G1</td>
</tr>
</tbody>
</table>

Results obtained by the SAP 2000 software

In order to compare the results of Plimit with those of commercial software, the two-story frame, shown above, was modeled using the SAP2000 software. This software allows plotting the resultant base shear according to the displacement of the control point, Node C (Fig. 2). This curve, also called capacity curve, shows that the limit load that the structure can support is about 531 KN (Fig. 4). This value corresponds to a 0.98 load factor.

The SAP2000 software can give the location of plastic hinges (Tab. 4). This hinges are formed in accordance with the increase of the load. The first plastic hinges appeared in A and H, corresponding to a load factor of 0.75. If this factor reached 0.89, the other two hinges are formed in G3 and B3 (Fig. 5).

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia Ixx</td>
<td>m^4</td>
<td>9.208 x 10^4</td>
</tr>
<tr>
<td>EA</td>
<td>KN</td>
<td>32.76 x 10^5</td>
</tr>
<tr>
<td>Elxx</td>
<td>KN.m^2</td>
<td>193.368 x 10^3</td>
</tr>
</tbody>
</table>

Table 1. Characteristics of the cross section

Figure 4. Curve capacity of the studied structure
Effect of St. Andrew’s cross on the load limit

The structure was calculated by SAP2000, but adding a St. Andrew’s cross on the ground floor (Fig. 6). 3 cross-sections were selected, which are TUBO D108X3.6, D127X4 and TUBO TUBO D159X4. The choice of three sections aims to realize not only the influence of the addition of this type of bracing, but also the influence of the cross section of the diagonals on the limit load.

The results obtained, for the first cross section (TUBO D108X3.6), show that the limit load increased by 148%, reaching a value of 1328 KN, instead of 534.2 KN (Fig. 7).

<table>
<thead>
<tr>
<th>Load factor</th>
<th>load (KN)</th>
<th>Number of hinges</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>0.75</td>
<td>402.9</td>
<td>2</td>
</tr>
<tr>
<td>0.89</td>
<td>479.0</td>
<td>4</td>
</tr>
<tr>
<td>0.98</td>
<td>531.1</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4. Sequence of formation of hinges

![Figure 5. Plastic hinges appeared for a load factor of 0.75 (left) and 0.89 (right)](image)

![Figure 6. Structure with St. Andrew cross](image)

![Figure 7. Capacity curves of the structure with St. Andrew cross](image)
10-storey building

The structure of a 10-storey building was modeled using 2000 SAP (Fig. 8). The height of each stage is of 3.00 m. The cross-section of the beams is IPE 600 while cross section of columns is TUBO 200x200x22,2.

The capacity curves obtained (Fig.9) shows that the overall strength of the structure in the direction (x) is greater than the direction (y), this is due to the big number of columns in the direction (x).

The SAP2000 software allows us to visualize the development of plastic hinges in the structure, including their location is shown in fig.10.
Discussion of results and conclusion

The capacity curves obtained by Plimit and SAP2000 are comparables (Fig. 6). They show that the point where a true change in the global stiffness of the frame is corresponding to a load factor of 0.89 (a value of the base shear of 479 KN). This point corresponds to the appearance of plastic hinges at the points A, H, B3, and G3.

If the load is further increased, the capacity curve reaches a plateau (point corresponding to a load factor of 0.98 and a resultant base shear of 531 KN). At this level of loading, the structure is substantially plasticized (damaged).

Regarding plastic hinge, the sequence of their formations, obtained by Plimit, is similar to that obtained by the SAP2000 software.

From the foregoing, we can conclude that the Plimit program gives reliable results.

With regard to the bracing system, the addition of a St. Andrew’s cross improves the behavior of the structure. Indeed, this change in structure leads to an increase of the limit load of 148%.

References

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