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Research Article

Comparative Study of Different Grid Generation Methods

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Abstract

The topic of grid generation has become a field on its own in the increasingly vast field of technology. Structured grid methods take their name from the fact that the grid is laid out in a regular repeating pattern called a block. These types of grids utilize quadrilateral elements in 2D and hexahedral elements in 3D in a computationally rectangular array. Although the element topology is fixed, the grid can be shaped to be body fitted through stretching and twisting of the block. Really good structured grid generators utilize sophisticated elliptic equations to automatically optimize the shape of the mesh for orthogonality and uniformity.

Keywords: Grid Generation, Topology, Algebraic Methods, Elliptic Methods, Hyperbolic Methods, Adaptive Grids, Structured Grid Generation Methods

Introduction

Below it is described a set of algebraic and differential methods that makes up the bulk of the available methods

Algebraic Methods

Algebraic methods are based on coordinate transformation equations in a physical domain. In their most simple form they are Lagrange and Hermite transformations (are called shearing transformations). Some methods are based on interpolation schemes in multi-dimensions. Transfinite interpolation (Eriksson, 1982) and multi-surface transformation (Eiseman, 1985) produce good grids for closed domains. Integration of the methods with additional control on the boundary values and elliptic smoothing (see further down) give efficient grid generation systems (for example ICEM/CFD, GridPro) These methods in their most developed form allow some control on the values of the derivatives at the boundary.

Elliptic Methods

Elliptic methods are based on the solution of elliptic partial differential equations with some conditions (called forcing terms) to force point bunching. The problem is formulated via a set of Poisson equations (Thompson, 1977) with forcing terms usually defined by the Thomas-Middlecoff terms (Thomas-Middlecoff, 1982), or by other appropriate control functions (Sorenson, 1995). 12 The solution of the system is iterative, for example with a Successive Over Relaxation (SOR) method. For large grids the computing time is considerable. Elliptic systems produce very smooth grids (sometimes too smooth) and they can be used to smooth out metric discontinuities in the transfinite interpolation systems (for this purpose also a Laplace smoother will suffice).

Hyperbolic Methods

Hyperbolic methods are based on the solution of partial differential equations of hyperbolic type that are solved marching outward from the domain boundaries. The idea of using hyperbolic PDEs is very effective for external flows where the wall boundaries (airfoil, wing, wing-body, etc.) are well defined, whereas the far field boundary is left arbitrary. This situation also eliminates the need to specify point distribution on some of the edges of the flow domain, and makes it handier than for example the transfinite interpolation methods. In its basic formulation (Steger-Chaussee, 1980) the hyperbolic grid generator is based on a condition of orthogonal, and a condition on the cell area. The method can be integrated with grid line smoothing and orthogonal checks.

Adaptive Grids

All the methods described above make use of some

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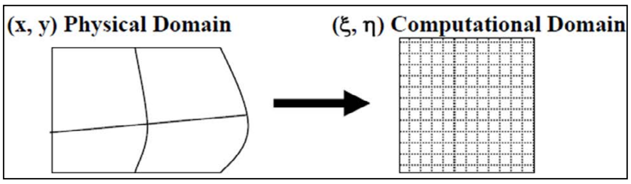
empirical knowledge about the form of the solution of the PDEs. This knowledge makes us force many points in regions of large field gradients (for ex. boundary layers). Better solutions could be obtained if a first guess grid could be adapted in a time marching numerical scheme to follow exactly the evolution of the field gradients (a particular difficult problem is the position of the shock wave in a transonic flow).

Deep Insight of Grid

Grid is a discrete Representation of the domain

Two views can be taken:

1. Mold a given two-dimensional domain in to a rectangle (and three-dimensional domain in to a box) by a suitable affine transformation



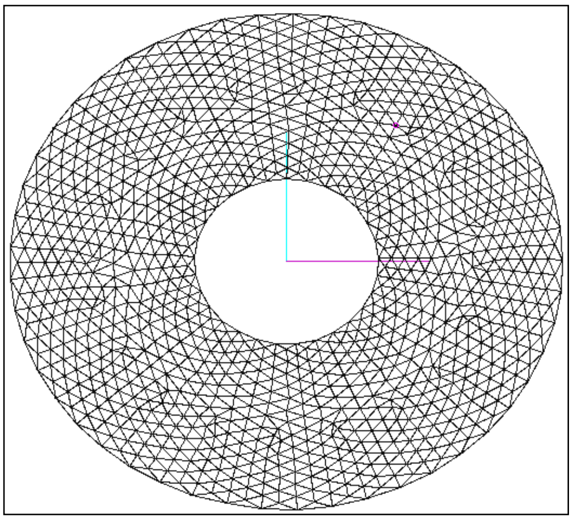
Find two functions f and g such that $\xi = f(x, y)$ and $\eta = g(x, y)$.

- Draw lines corresponding to constant values of $\xi = i\ \Delta\xi = i$ and $\eta = j\ \Delta\eta = j$

  for i = 1 to i max and j = 1 to j max in (x, y) plane.

- Intersection of these points gives (a) grid points (i, j) and also (b) quadrilateral cells.

2. Fill a given domain with simple shapes such as triangles (say) so that the given domain is fully covered.



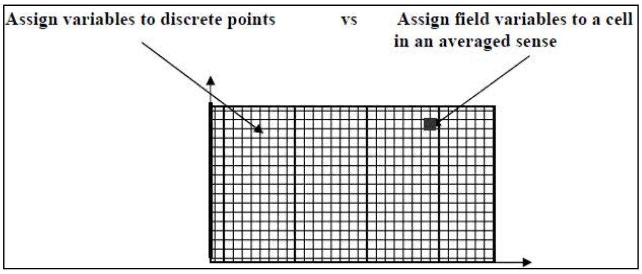
- Domain here is space between two circles. The simple shape chosen to fill the space is triangle.
- Triangles collectively cover the complete space, triangles do not intersect each other

Thus Grid (in 2D) is defined by

A set of discrete points and a set of cells (usually quadrilaterals or triangles) Collection of neighboring discrete points forming *a set of cells* and Lines extending connecting the discrete points, i.e. *edges of cells.* If the edges span the across the domain and join the same number of grid points they are called *grid lines.*

Grid also helps in representing a field in discrete form

- Represents continuous variables only at finite number of points
- Helps in converting PDE to FDE, FVE or FEE
- Solution of these equations gives field at discrete points.



Representation of space and field can be done in two ways.

Finite Difference method (FDM):

- Field assigned to grid points only
- Variation of the field in between the points is not explicitly defined
- Resulting equations satisfy only at grid points

Finite Element Method (FEM):

- Field assigned to grid points and is well as some intermediate points
- Variation of the field in between the points is explicitly defined
- Resulting equations satisfy in overall sense

Finite Volume Method:

- Field assigned to cells and it is assumed to be constant in a cell
- Resulting equations is conservation in every cell

Importance of Grid Generation

The topic of grid generation has become a field on its own in the increasingly vast field of computational fluid dynamics. Some general considerations regarding suitable methods and their role in computational fluid dynamics are the following:

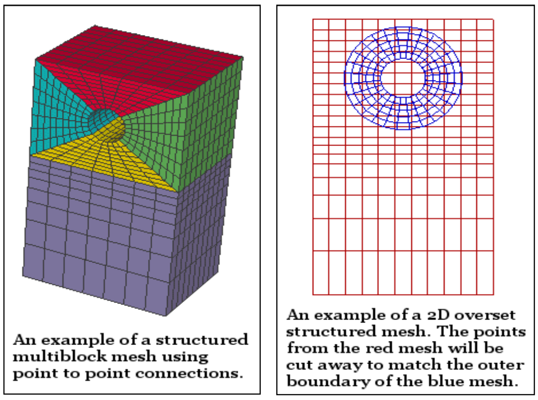
- Almost any method works on a good grid, whereas the bad methods only work on good grids.
- If one had enough resolution (e.g. enough points), then the grid quality would be of minor importance,

provided that some basic requirements are satisfied. If the grid is necessarily coarse, then its quality becomes essential.

- A good grid can accelerate the convergence of the solution, while a *bad* grid can even lead to a divergent iteration history.

Structured Grid Generation Methods

Structured grid methods take their name from the fact that the grid is laid out in a regular repeating pattern called a block. These types of grids utilize quadrilateral elements in 2D and hexahedral elements in 3D in a computationally rectangular array. Although the element topology is fixed, the grid can be shaped to be body fitted through stretching and twisting of the block. Really good structured grid generators utilize sophisticated elliptic equations to automatically optimize the shape of the mesh for orthogonality and uniformity.



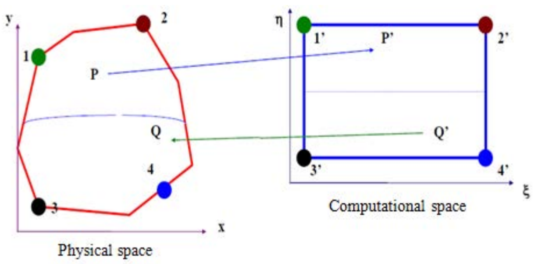
Typical 3D and 2D structured mesh

Structured meshes lead to very efficient numerical methods, High quality sufficiently simple geometries, large grid control when high anisotropy is required, Multi-Block approach allows for realistic geometries.

Two Dimensional Structured Grids

Structured grids are generated by a mapping the Physical Domain to a Computational Domain 2-D domain, say (x y plane or physical plane) or a surface in (x, y, z) space can be mapped to on a rectangle in say (ξ, η) plane (computational domain).

Mathematically $\xi = F(x, y)$, $\eta = G(x, y)$ or $x = H(\xi, \eta)$, $y = I(\xi, \eta)$ this mapping to be one to one mapping



The Domain acquired Rectangular topology; i.e.

The boundary of the domain got divided in to four separate parts each getting mapped to sides of rectangle in plane

- Four points on the boundary got explicitly mapped to four vertices
- If m lines are drawn along ξ axis and n lines are drawn along η axis, there will be m x n lines in the domain
- The intersection of these lines will generate grid points. These lines also divide the domain in to cells (m-1) x (n-1) number

Thus a structured grid gets generated automatically if (F and G) or (H and I) are found out.

Numerical method where in discrete points (X_{ij}, Y_{ij}) for i = 1 to m and j = 1 to n defined as

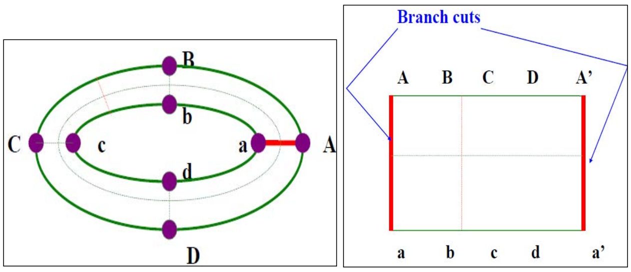
$X_{ij} = H(\xi_{ij}, \eta_{ij})$ for i = 1 to m and j = 1 to n

$Y_{ij} = I(\xi_{ij}, \eta_{ij})$ with ξ_{ij}, η_{ij} are equally spaced . This Procedure is called automatic numerical grid generation.

Concepts of Topology:

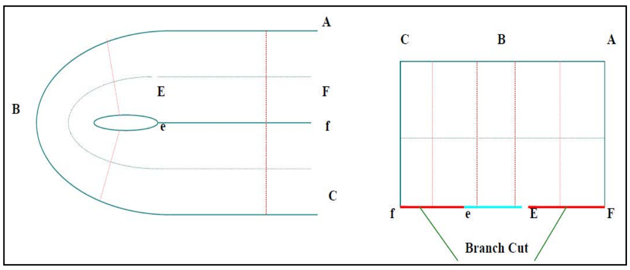
Topology of Grids in Doubly Connected Region:

O-Topology:



- Points on both boundaries is the same
- Flow gets resolved better on the internal surface
- Good for Simulation of in viscid flows
- Require Periodic boundary conditions

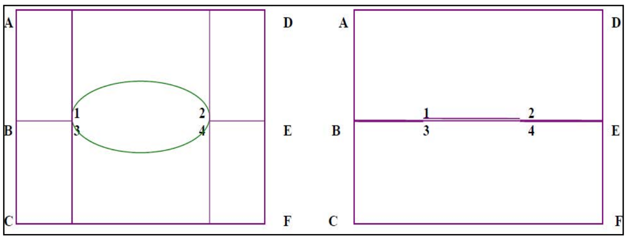
C-Topology



- No. of Points on the outer boundary is equal to the number of grid points on the airfoil + wake.
- Good for Simulation of viscous flows
- Requires a Special Provision for implementation of Boundary Conditions.

Topology of Grids in Multiple Connected Regions

H-Topology



- No. of Points on the object immersed in the domain is much less
- Easy to generate
- Good for Simulation of cascade flows
- Flow near Leading Edge does not get Resolved
- Boundary condition is used in the Domain

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