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Research Article

Optimum Design of a Spring-Loaded Linkage Mechanism in the Presence of Friction for Static Balancing

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ABSTRACT

The influence of friction in a spring-assisted automotive hood mechanism in the presence of joint friction, is investigated. The objective is to optimally introduce joint friction to keep the hood opened in static equilibrium over a wide range of positions. Several criteria have been investigated to optimally determine the spring parameters, their attachment points as well as the friction at the joints, while minimizing the opening and closing force of the hood.

Keywords: Static Balancing, Friction, Spring-Loaded Mechanism, Optimum Design

Introduction

The presence of friction is usually considered undesirable in machinery. There are however situations in machinery where friction is actually desirable. These situations occur when friction assists in allowing the mechanism to maintain static equilibrium, or for safety consideration.[3,4]

Many studies have been conducted to maintain static equilibrium in a mechanism under an applied force such as spring-assisted band drives and linkage mechanisms.[2-5] This article examines one such linkage mechanism, with springs, to achieve static equilibrium over a wide motion range of the linkage. With optimization, various parameters such as attachment locations of the spring and its stiffness may then be determined. While the procedure seems straightforward, in reality the results are highly dependent on the specification of the optimally criterion in conjunction with reasonable or practical constraints on the parameters to be optimized. While many articles have been published on the application of optimization to mechanisms, such as,[6,7] this article applies optimization to the design of a spring-loaded mechanism in the presence of joint friction for the purpose of achieving static balancing.

The Problem: Static Equilibrium of a Spring-Loaded Linkage Mechanism with Joint Friction

This investigation examines a hood-linkage with an extension spring to statically support the weight of an automotive hood. There are two configurations. In Figure 1a, points *A, C, D* and *E* are the revolute joints of the four-bar linkage with the hood *CP* being part of the coupler link *CD*. Attachment points as well as the spring stiffness and preload, are to be optimally determined. In this Figure (Configuration 1), the extension spring attachment point *B*, is on the first link *AC*, while in Figure 1b, the spring attachment *B* is instead on the second link *DE* (Configuration 2). Friction torque is introduced in each of the revolute joints. The friction torques can be specified, or optimally determined in a manner that minimizes the peak applied vertical force at point *P*, in opening and closing the hood. In the presence of spring assist, the lifting force varies and may even change direction as the hood travels from a closed to an opened position.

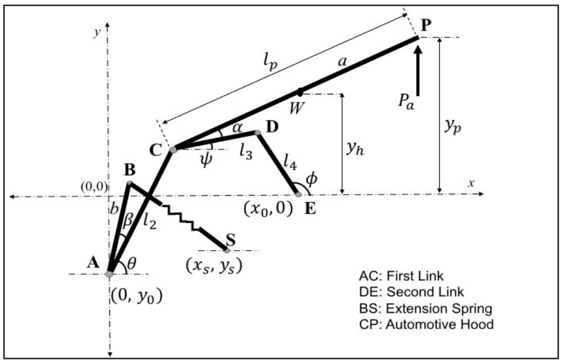

Figure 1a.Hood Linkage with Spring-Assist on the First Link (Configuration 1)

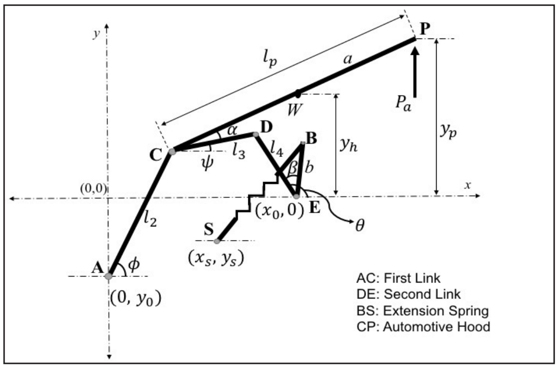

Figure 1b.Hood Linkage with Spring-Assist on the Second Link (Configuration 2)

An in-depth study of this linkage with frictionless joints, has been conducted.[1] The results show that it is possible for the automotive hood to be at statically equilibrium conditions at only two positions. In other words, the hood will tend to open or to close at all other positions, thereby necessitating either a downward (negative P_a) or an upward (positive P_a) to keep the hood in static equilibrium as the case may be. It may therefore seem reasonable to introduce friction at the joints so as to abrogate any need for an externally applied vertical force P_a to keep the hood in static equilibrium.

The question is how large a friction torque is needed at the joints of the linkage to keep the hood in static equilibrium over the entire motion range while keeping the peak opening and closing forces to a minimum. This term, “minimum” needs to be precisely defined. In this article, several optimally criteria will be presented, that attempts to minimize the applied peak force on the hood, while optimizing the spring parameters, their attachment locations as well as the level of frictional torque to be applied at the linkage joints.

The choice of an optimality criterion plays a crucial role in achieving a practical outcome that conforms to the constraints and other requirements of the application, requirements that are not immediately obvious when formulating such a design optimization problem.

Optimization Criteria

In a previous article,[1] the objective is to minimize the vertical lifting force P_a on an automobile hood with a spring-loaded linkage with frictionless joints. To maintain static equilibrium, as the hood opens, the applied force changes from positive (upward) to negative and then back to positive. To minimize the peak applied force while accounting for sign changes, the objective function is posed in a quadratic form: $\int P_a{}^2 d\theta$ where θ is the hood angle relative to the horizontal.

In this article, two additional objective functions are also investigated. The goal is to arrive at balanced peak force fluctuations so as reduce the level of friction needed to overcome these fluctuations, when friction in the joints are subsequently introduced. Three different objective functions are considered in the absence of friction at the joints and the results are shown in Figs. 2a and 2b for Configurations 1 and 2, respectively.

Objective Function I: $\int P_a{}^2 d\theta$

With Configuration 1 (spring assist at the first link), the blue curve in Figure 2a, shows that negative and positive force fluctuations about zero are biased toward the positive so that the peak (28 *N*) is much larger than negative peak force (7 *N*). This is undesirable. Figure 2b shows the case for Configuration 2 (spring-assist at the second link).

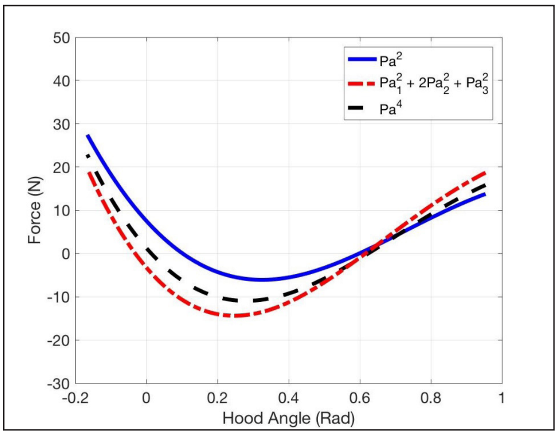

Figure 2a.Applied Force for Configuration 1

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The positive peak (12 *N*) is still more than twice that of the negative peak at 5 *N*. Overall, the forces fluctuate much less than that of the first configuration.

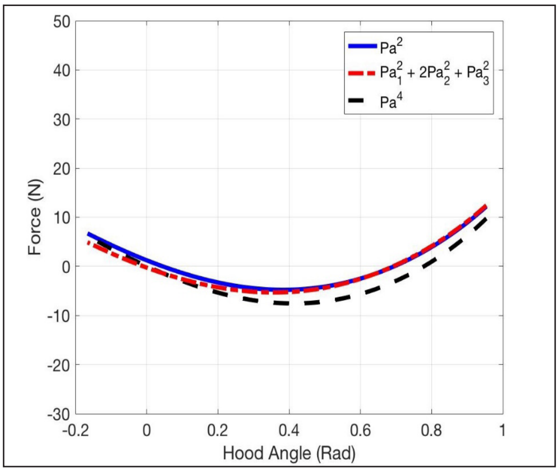

Figure 2b.Applied Force for Configuration 2

Objective Function II: $\int(P_{a_1}{}^2 + 2P_{a_2}{}^2 + P_{a_3}{}^2)d\theta$

The second form of the objective function is an attempt to address the situations observed in Objective Function I. Suppose P_{a1}, P_{a2}, P_{a3} are the forces when hood is at the fully opened position, the intermediate position and the closed position, respectively. The red-dashed curves in Figs. 2a and 2b show that the force peak points at the closed, opened and intermediate points during hood opening are more balanced result though, in some positions, the peak forces are higher than those in Objective Function I.

Objective Function III: $\int P_a{}^4 d\theta$

This objective function is presented in an attempt to severely penalize peak forces. The black dashed curves in Figs. 2a and 2b show that among these three objective functions, this third form results in balanced peak forces and is therefore selected as the objective function in this investigation. Based on this Objective Function III, optimization results for the design case with frictionless joints, are shown in Tables B1 and B2 in Appendix B. In each of these spring designs, attachments locations to each of the two oscillating links of the hood are presented. In comparing the link at which the extension spring is loaded, Figure 2b shows that the peak lifting force on hood is the lowest with extension-spring assist at the second link (Configuration 2).

Introducing Friction at the Joints

Having determined a suitable objective function, we can now proceed to include joint friction into the design process. There are four revolute joints in this hood linkage and coulomb friction is assumed at these joints. When the engine hood opens and closes, coulomb friction in the joints operate differently for the two motions and therefore, it is crucial to correctly determine the direction of the joint friction torques. We begin by writing the virtual work equation for the linkage mechanism that includes joint friction.

Virtual Work Equation for the Hood Linkage

The virtual work equation for the linkage in the presence of an extension spring attached to the first link (Configuration 1) as shown in Figure 1a, is given by:

$$\delta W = P_a \delta y_p - \delta V_s - \delta V_h \pm F_{f_A}\delta\theta \mp F_{f_C}(\delta\psi - \delta\theta)$$
$$\mp F_{f_D}(\delta\phi - \delta\theta) \pm F_{f_E}\delta\phi \quad (1a)$$

where δW is the virtual work, P_a is the force applied to open or to close the hood. δy_p is the vertical virtual displacement of the force applied at the front end of the hood. δV_s and δV_h are conservative potentials of the springs and the hood respectively. $F_{f_A}, F_{f_C}, F_{f_D}$ and F_{f_E} are friction torques at joints *A, C, D* and *E* respectively. The positive sign in the is applied when the effective friction direction is upward and negative for a downward friction force. Therefore, the friction forces, F_{f_A}, F_{f_E} are positive and F_{f_C}, F_{f_D} are negative when the hood is opening and F_{f_A}, F_{f_E} are negative and F_{f_C}, F_{f_D} are positive when the hood is closing. Based on this, friction torques are positive if there are directed in a counter-clockwise (CCW) direction. From Eq. (1a), we have:

$$\frac{dW}{d\theta} = P_a\frac{dy_p}{d\theta} - \frac{dV_s}{d\theta} - \frac{dV_h}{d\theta} \pm F_{f_A} \mp F_{f_C}\left(\frac{d\psi}{d\theta} - 1\right)$$
$$\mp F_{f_D}\left(\frac{d\phi}{d\theta} - 1\right) \pm F_{f_E}\frac{d\phi}{d\theta} \quad (1b)$$

A similar set of equations for Configuration 2 (Figure 1b) are given by Eqs. (A1) and (A2) in Appendix A. In static equilibrium, virtual work must vanish, i.e.,

$$\delta W = 0 \quad (2)$$

In this article, all four joints are subjected to joint friction at the same time, without constraints on the friction magnitude at each joint. This procedure allows the optimization process to determine the necessary optimal friction at each joint, along with all the other parameters relating to the spring, simultaneously. The presence of friction generally increases the required force to open or close the hood and must also be sufficiently large to support the hood at any opened position.

The relationship between θ with ψ and ϕ in Eq. (1b), for Configuration 1 is given by the kinematic loop equation for a four-bar linkage:

$$l_2 e^{i\theta} + l_3 e^{i\psi} - l_4 e^{i\phi} - x_0 - iy_0 = 0 \quad (3)$$

where l_2, l_3 and l_4 are the lengths of links *AC, CD* and *DE* respectively; $(0, y_0)$ and $(x_0, 0)$ are the coordinates of joints *A* and *E* respectively. From Eq. (3), ϕ and ψ can then be

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solved as a function of θ from which, y_p, V_s and V_h can readily be determined.

Note that the hood opening and closing forces are different in the presence of joint friction. Therefore, there are two objective functions for minimizing the applied force , one for opening and the other for closing the hood. Opening and closing objective functions are given by Eq. (4) and (5) respectively:

$$f_1 = \int (P_{a_o})^4 d\theta \quad (4)$$

$$f_2 = \int (P_{a_c})^4 d\theta \quad (5)$$

To minimize these opening and closing peak forces the following function can be used to bring about an optimization combining the aforementioned objective functions:

$$\text{minimize } f = w_1 \int (P_{a_o})^4 d\theta + w_2 \int (P_{a_c})^4 d\theta \quad (6)$$

where w_1 and w_2 are weights for the opening and closing forces, P_{a_o} and P_{a_c} respectively.

Problem Formulation

The problem specification shown below is for Configuration 1 (Figure 1a) in the presence of joint friction at each joint, that is to be determined by the optimization as design variables. The objective function given by Eq. (6) above, is subjected to the following boundary conditions and constraints.

Boundary conditions:

$$\left.\begin{array}{c} 0.1 < x_1 < 10^6 \\ -1.98\pi < x_2 < 1.98\pi \\ 0.1 < x_3 < 0.5 \\ 0.001 < x_4 < 0.9 \\ -0.5 < x_5 < 0.5 \\ -0.4 < x_6 < 0 \\ 0 < x_7 < 10^5 \end{array}\right\} \quad (7)$$

Constraints:

$$\left.\begin{array}{c} x_4 < l_s \\ 0.2 < P_{a_o} \end{array}\right\} \quad (8)$$

where $x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 are respectively k (spring constant); β (angle between links *AB* and *AC* or between links *DE* and *EB* for Configurations 1 and 2 respectively); b (length of link *AB* or *DE* for Configurations 1 and 2 respectively); l_o (unloaded spring length); x_s and y_s (*x* and *y*-axes coordinates of spring attachment point on ground); and F_f (joint friction).

The constraint $x_4 < l_s$ ensures that extension spring does not change into compression spring during hood motion and the constraint $0.2 < P_{a_o}$ keeps opening force positive.

Result

The result of the optimization procedure is shown in Figure 3a for Configuration 1. The applied force, P_a, is plotted as a function of hood angle with the bold line showing the case for frictionless joints. The dashed upper curve is that for opening the hood, while the dotted curve, is for closing. When the required force is in the negative region, the hood tends to open itself and joint friction should act in the opposite direction to keep hood in static equilibrium. Similarly, when the required force is in positive region, the hood tends to close itself and joint friction can be introduced to oppose that motion to keep it static.

The optimization has determined the requisite friction at the joints so that the applied force to open the hood (given by the dashed curve) is entirely positive. Similarly, that joint friction must also resist hood closing. This requires a negative applied force (downwards) for a large portion of hood closing except for hood angle less than approximately 0.05 rad. Such a force characteristic is actually advantageous in an automotive hood application and the reason for this is explained in the Discussion section.

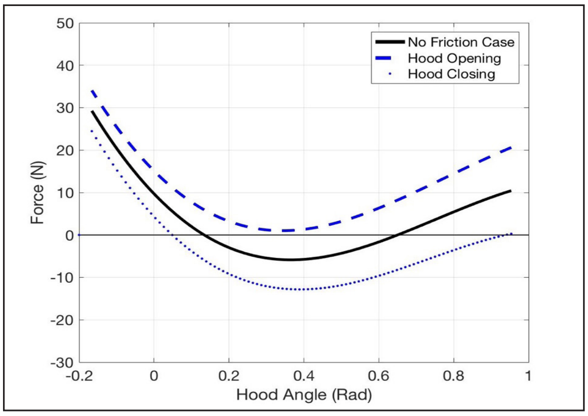

Figure 3a.Optimized Opening and Closing Forces for Configuration I

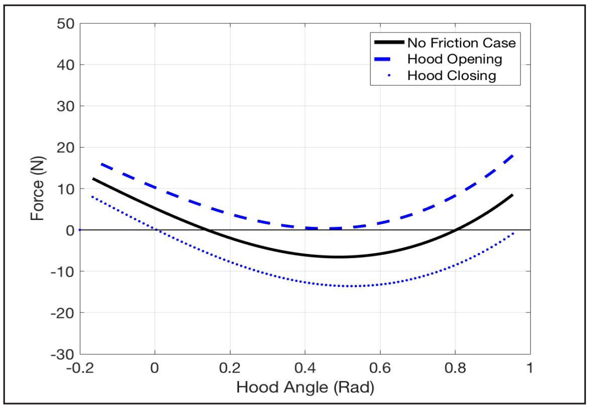

Figure 3b.Optimized Opening and Closing Forces for Configuration 2

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Similarly, for Configuration 2, the results for the opening and closing forces are shown in Figure 3b. Comparing the two configurations, the basic shapes of the results are similar except that the spring-assist on the second link configuration gives lower opening and closing forces.

Numerical results for both configurations are summarized in Table 1. In the first four rows, the values are the friction permitted in only that one joint in the linkage. In row five, a constraint is specified that all four joints are to have the same friction and in the last row, the joint frictions shown are the optimum for that spring-assist link configuration.

Table 1.Joint Friction Results for the Two Spring-Assist Configurations

Joint Friction (Nm/rad)	Configuration 1	Configuration 2
F_{f_A}	5.844	7.735
F_{f_C}	1.773	2.47
F_{f_D}	1.283	2.617
F_{f_E}	1.836	4.349
$F_{f_A} = F_{f_C} = F_{f_D} = F_{f_E}$	0.554	1.001
$F_{f_A} \neq F_{f_C} \neq F_{f_D} \neq F_{f_E}$	$F_{f_A} = 5.844$ $F_{f_C} = F_{f_D} = F_{f_E} = 0$	$F_{f_C} = 2.546$ $F_{f_A} = F_{f_D} = F_{f_E} = 0$

Note that in Figs. 3a and 3b, the optimization determines the optimum friction needed at each joint and the values are shown in Row 6 in Table 1 above. Suppose that joint friction is not included in the optimization but is subsequently added equally to each of the four joints until the opening force is all positive. This case has been studied and the results are shown in Figs. C1 and C2, in Appendix C. A comparison between Figs. 3a and C1, Figs. 3b and C2 shows that results are very similar except that when joint friction is optimized, slightly lower applied forces are required.

In the following section, a detailed discussion of how the applied opening and closing forces under joint friction for static equilibrium of the hood is provided.

Discussion: Effects of Friction on the Applied Force for Static Equilibrium of the Hood

In the Figure 4, the solid line 1-2-3-4 shows the force needed to keep the hood in static equilibrium with frictionless joints. The line segment 1-2 is positive as is from 3 to 4. These line segments show that an upward force, is needed to keep the hood in static equilibrium while a negative force (downward) from 2 to 3 is needed. Note that at point 1, the hood is at its full open position while at point 4, it is closed. This force variation in maintaining static equilibrium can be split into three regions *A, B* and *C* as shown in the Figure

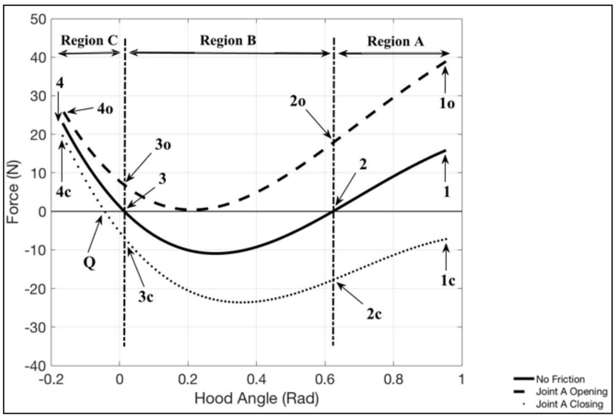

Figure 4.Friction at Joint A only for Configuration 1

When coulomb friction is added to any or all of the linkage joints, two curves result. The upper dashed curve given by 4o-3o-2o-1o shows the required force needed to open the hood, while the lower dotted curve 1c-2c-3c-4c is for closing it. Note the curve numbering order for the opening and closing curves. When the direction of motion changes, so does friction, thereby resulting in these two different applied forces. An explanation of how these curves are relevant to whether the hood stays in static equilibrium in the presence of coulomb friction in the joints is provided below. The entire hood motion is divided into three regions.

Region A

As previously noted, in a frictionless condition the curve 1-2 infers that an upward force is needed to keep the hood is static equilibrium. Therefore, in the absence of an applied upward force, the hood tends to close. In the presence of joint friction, to close the hood one needs to apply a negative (or downward) force along 1c-2c. In this Region A, since the curve 1c-2c is negative, coulomb friction is more than sufficient to keep the hood in static equilibrium because a negative 1c-2c curve denotes a downward force that is needed to overcome this friction when closing the hood in this region. The positive curve 2o-1o shows that the opening force in the presence of friction is higher than the frictionless case.

Region C

The segment 3c-Q is exactly the same as in Region A, wherein a negative (downward) force is needed to close the hood. Here, friction in the joints is sufficient to keep the hood in static equilibrium. There is however a slight difference from Region A in that the segment Q-4c in this Region C is positive. This implies that since a positive (upward) force is needed to support the hood while it is closing, joint friction in this segment is insufficient to keep the hood in static equilibrium. In this particular application of an automotive hood, it turns out to be an advantage. When the hood is near or at the closed position, that is a safety feature; the hood tends to remain closed to prevent

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it from springing open on its own. In fact, the positive curve 4o-3o shows that friction actually makes it even less likely should the hood-latch malfunctioned.

Region B

In this region, in the absence of friction, the curve 3-2 shows that a negative (downward) force is needed to keep the hood in static equilibrium which means that the hood would spring open without an externally applied negative (downward) force. The positive curve 3o-2o implies that with joint friction, an upward force is needed to open the hood. If this upward force were not applied, joint friction is sufficient to keep the hood from springing open; the hood will be in static equilibrium with just joint friction. The curve 2c-3c shows that a higher downward force is required to close the hood in this region when compared to the frictionless case.

The general requirements for a mechanical system to be in static equilibrium is that the lower dotted curve 1c-4c has to be entirely negative. This ensures that friction at the joints is sufficient to keep the system in static equilibrium.

Conclusion

This article uses an automotive hood linkage as an example to design into a spring-assisted linkage, joint friction so as to achieve static equilibrium. An optimization procedure is used to determine spring attachment locations and spring characteristics, as well the optimum friction allocation to the joints to minimize the opening and closing forces for the hood, while achieving static equilibrium of the hood. It was found that a suitable objective function is crucial to obtain a balanced force fluctuation during the opening and closing hood motion.

Two tension-spring attachment configurations were investigated, one on the first and the other on the second link. The results show that lower opening and closing forces occur for a spring-assist at the second link. To ensure static equilibrium of the entire motion of the hood, the applied opening force must be positive (upwards) and the closing force is negative (downwards) for the entire range of motion of the hood.

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Appendix

- **Appendix A**

Virtual Work Equations for Configuration 2:

$$\delta W = P_a \delta y_p - \delta V_s - \delta V_h \pm F_{f_A} \delta\phi \mp F_{f_C}(\delta\psi - \delta\phi) \mp F_{f_D}(\delta\theta - \delta\psi) \pm F_{f_E} \delta\theta \qquad \text{(A1)}$$

and

$$\frac{dW}{d\theta} = P_a \frac{dy_p}{d\theta} - \frac{dV_s}{d\theta} - \frac{dV_h}{d\theta} \pm F_{f_A} \frac{d\phi}{d\theta} \mp \frac{F_{f_C}(d\psi - d\phi)}{d\theta} \mp F_{f_D}\left(1 - \frac{d\psi}{d\theta}\right) \pm F_{f_E} \qquad \text{(A2)}$$

- **Appendix B**

Table B1.Optimum extension spring results for Configuration 1 (Frictionless joints)

Objective Function	k (N/m)	β (rad)	b (m)	l_0 (m)	x (m)	y (m)
$\int P_a{}^2 d\theta$	312013.536	-1.163	0.046	0.708	0.7	-0.395
$\int P_a{}^4 d\theta$	661818.814	-1.149	0.037	0.691	0.69	-0.344
$\int (P_{a_1}{}^2 + 2P_{a_2}{}^2 + P_{a_3}{}^2) d\theta$	103456.655	5.045	0.098	0.655	0.7	-0.4

Table B2.Optimum extension spring results for Configuration 2 (Frictionless joints)

Objective Function	k (N/m)	β (rad)	b (m)	l_0 (m)	x (m)	y (m)
$\int P_a{}^2 d\theta$	34111.038	4.564	0.028	0.398	0.7	-0.4
$\int P_a{}^4 d\theta$	111042.123	2.852	0.014	0.9	-0.476	-0.4
$\int (P_{a_1}{}^2 + 2P_{a_2}{}^2 + P_{a_3}{}^2) d\theta$	17230.304	2.744	0.042	0.845	-0.5	-0.4

- **Appendix C**

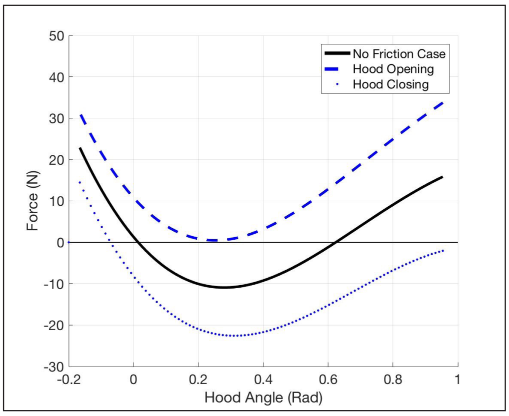

Figure C1.Subsequent Addition of Joint Friction for Configuration 1

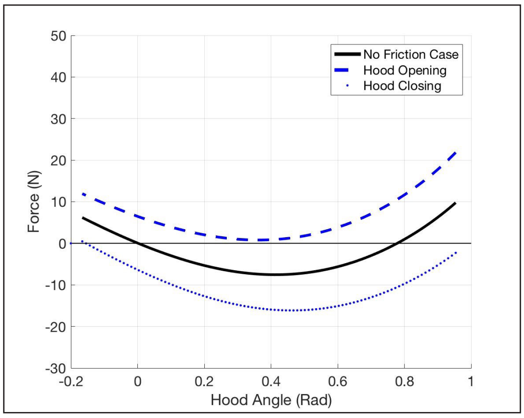

Figure C2.Subsequent Addition of Joint Friction for Configuration 2

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