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Article

Higher Order Reduction of Uncertain Systems: Affine Arithmeti

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I N F O

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A B S T R A C T

In this paper the Modified Routh Approximation (MRA) and Affine Arithmetic (AA) methods are investigates for obtaining the reduced order model (ROM) of SISO, discrete & MIMO uncertain systems into lower order system. Rigorous study and analysis of physical system direct to the outcome of systems with uncertainty instead of certain coefficients. Thus, systems having uncertain but bounded parameters known as uncertain systems are under consideration in this paper. An effective algorithm to determine the reduced order model is proposed here. This proposed methodology is verified using numerical examples available from the literature.

Keywords: Continuous, Discrete & MIMO Uncertain Systems, Modified Routh Approximation Method, Affine Arithmetic

Introduction

The analysis and design of practical control systems become complex when the order of the system increases. Therefore, to analyze such systems, it is necessary to reduce it to a lower order system, which is a sufficient representation of the higher order system. From last few decades, model order reduction problem has been ample area of research since it minimizes computational efforts and increases the possibility of accurate simulation and design process. Different model order reduction methodologies have been proposed for reduction of fixed coefficient systems, SISO continuous,[1-2] and discrete interval systems[3-4] MIMO systems.[5-6]

Affine arithmetic is potentially useful in every numeric problem where one needs guaranteed enclosures to smooth functions, such as solving systems of non-linear equations, analyzing dynamical systems, integrating functions differential equations, etc. Affine arithmetic is one of several computational models that were proposed to overcome the problem of interval arithmetic. But, affine arithmetic keeps track of first order correlations between computed and input quantities; these correlations are automatically exploited in primitive operations, with the result that in many cases affine arithmetic is able to produce interval estimates that are much better than the ones obtained with standard interval arithmetic. Moreover, affine arithmetic also implicitly provides a geometric representation for the joint range of related quantities that can be exploited to increase the efficiency of interval methods. Like interval arithmetic, affine arithmetic keeps track automatically of the round-off and truncation errors affecting each computed quantity.

This paper is organized as follows: Reduction procedure is given in Section II. Numerical examples and comparison of proposed method with other well known techniques is Shown in Section III and conclusion and references in Section IV.

Reduction Procedures

SISO Uncertain Continuous System

In this section, a simple method for reducing higher order is explained. The coefficients of reduced order are obtained by using Modified Routh Approximation Method, without finding time moments of the original system. In this method Gamma and Alpha table is required to be formulated.

The higher order continuous system transfer function is given as follows:

$$G(s)=\frac{B_1+B_2\,s+B_3\,s^2\ldots\ldots.+B_{n-1}\,s^{n-1}}{A_1+A_2\,s+A_3s^2+\cdots\ldots+A_{n-1}s^{n-1}+A_ns^n} \quad (1)$$

The transfer function G(s), expands as follows:

$$G(s)=\left[\frac{\gamma_1}{s}+\beta_1\right]f_1(s)+\left[\frac{\gamma_2}{s}+\beta_2\right]f_1(s)f_2(s)+\ \ldots\ldots + \left[\frac{\gamma_{\left[\frac{n+1}{2}\right]}}{s}+\beta_{\left[\frac{n+1}{2}\right]}\right]f_1(s)f_2(s)\ldots.f_{\left[\frac{n+1}{2}\right]}(s) \quad (2)$$

Where, f_k (s), k=1, 2...... [(n+1)/2] are determined by following continued fraction:

$$G_k(s)=\frac{1}{\gamma_k s^{-1}+\alpha_k s}+\frac{1}{\gamma_{k+1}s^{-1}+\alpha_{k+1}s}+\cdots+\frac{1}{\gamma_{\left[\frac{n+1}{2}\right]}s^{-1}+\alpha_{\left[\frac{n+1}{2}\right]}s}$$

K=1,2,3....[(n+1)/2] (3)

SISO Discrete Uncertain System

Consider the kth order single input single output discrete time interval system as follows:

$$G(z)=\frac{[u_1^-,u_1^+]z^{k-1}+[u_2^-,u_2^+]z^{k-2}+\ldots+[u_k^-,u_k^+]}{[v_0^-,v_0^+]z^k+[v_1^-,v_1^+]z^{k-1}+\ldots+[v_k^-,v_k^+]}=\frac{\Sigma[u_i^-,u_i^+]z^{k-i}}{\Sigma[v_j^-,v_j^+]z^{k-j}}=\frac{A_k(z)}{B_k(z)} \quad (4)$$

Where, i=1,2,3,.....k and j=1,2,3,......k are the interval parameters.

Then mth order reduced system may be obtained as follows

$$T_m(z)=\frac{[x_1^-,x_1^+]z^{m-1}+[x_2^-,x_2^+]z^{m-2}+\cdots+[x_m^-,x_m^+]}{[y_0^-,y_0^+]z^m+[y_1^-,y_1^+]z^{m-1}+\cdots+[y_m^-,y_m^+]}=\frac{\Sigma[x_i^-,x_i^+]z^{m-i}}{\Sigma[y_j^-,y_j^+]z^{k-j}}=\frac{A_m(z)}{B_m(z)} \quad (5)$$

Where i=1,2, 3,.....m and j=1,2,3,......m are the interval parameters.

MIMO Uncertain System

Let the original high order q-input, p-output MIMO interval system be given in transfer matrix form as:

$$\mathbf{G}_n(s)=\frac{[\mathbf{N}_{i,j}(s)]}{D_n(s)};\text{with}\ i=1,2,3,\ldots,p;\ j=1,2,3,\ldots,q\ \text{and}(m\le n)$$

$$=\begin{bmatrix} N_{11}(s) & N_{12}(s) & \ldots & N_{1q}(s)\\ N_{21}(s) & N_{22}(s) & \ldots & N_{2q}(s)\\ \ldots & \ldots & \ldots & \ldots\\ N_{p1}(s) & \ldots & .. & N_{pq}(s)\end{bmatrix} \quad (6)$$

where the common denominator polynomial is

$$D_n(s)=[A_n^-,A_n^+]s^n+[A_{n-1}^-,A_{n-1}^+]s^{n-1}+\ldots\ldots\ldots.+[A_1^-,A_1^+]s+[A_0^-,A_0^+]$$

and the scalar numerator polynomials are for i = 1, j = 1,

$$N_1(s)=[B_m^-,B_m^+]_1\ s^m+[B_{m-1}^-,B_{m-1}^+]_1\ s^{m-1}+\ldots\ldots\ldots+[B_1^-,B_1^+]_1\ s+[B_0^-,B_0^+]_1$$

for i = 1, j = 2,

$$N_2(s)=[B_m^-,B_m^+]_2\ s^m+[B_{m-1}^-,B_{m-1}^+]_2\ s^{m-1}+\ldots..+[B_1^-,B_1^+]_2\ s+[B_0^-,B_0^+]_2$$

and in general,

$$N_j(s)=[B_m^-,B_m^+]_j\ s^m+[B_{m-1}^-,B_{m-1}^+]_j\ s^{m-1}+\ldots\ldots\ldots.+[B_1^-,B_1^+]_j\ s+[B_0^-,B_0^+]_j$$

It is proposed to obtain a low order model for the above original high order interval system as:

$$R_r(s)=\frac{[n_{i,j}(s)]}{d_r(s)};\text{with}\ \ i=1,2,3,\ldots.,p\quad j=1,2,3,\ldots.,q$$

$$=\begin{bmatrix} n_1\ (s) & n_2\ (s) & \ldots & n_{1q}(s)\\ n_2\ (s) & n_2\ (s) & \ldots & n_{2q}(s)\\ \ldots & \ldots & \ldots & \ldots\\ n_{p1}(s) & \ldots & \ldots & n_{pq}\ (s)\end{bmatrix} \quad (7)$$

Numerical Example and Results

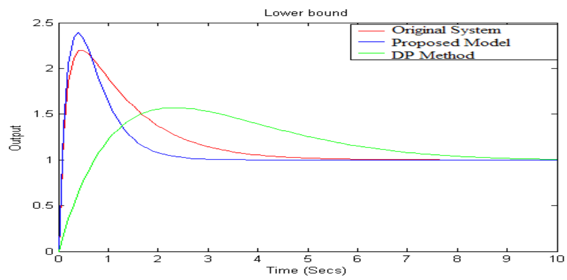
Case 1: SISO uncertain continuous system:

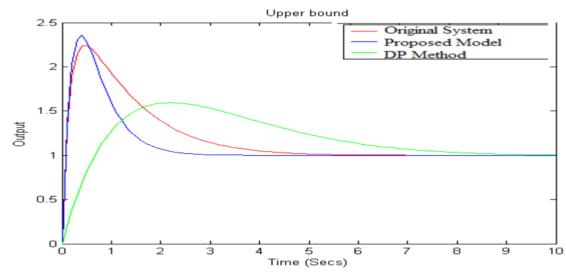
Consider an 8th order stable system given by

$$G(s)=\frac{[18,19.2]\,s^7+[514,529.5]\,s^6+[5982,6161]\,s^5+[36382,37473]\,s^4+[122664,126344]\,s^3+[222088,228751]\,s^2+[185760,191333]\,s+[40320,41530]}{[1.0,1.05]\,s^8+[36,36.5]\,s^7+[546,551.5]\,s^6+[4536,4581]\,s^5+[22449,22673.5]\,s^4+[67284,67956.5]\,s^3+[118124,119305]\,s^2+[109584,110680]\,s+[40320,41530]}$$

For the above original system, the 2nd order model by the proposed method is obtained as given below:

$$R_2(s)=\frac{[1.9402152,2.0920942]\,s+[0.4169617,0.4586435]}{s^2+[1.1445765,1.2102093]\,s+[0.4169617,0.4586435]}$$





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SISO Discrete Uncertain System

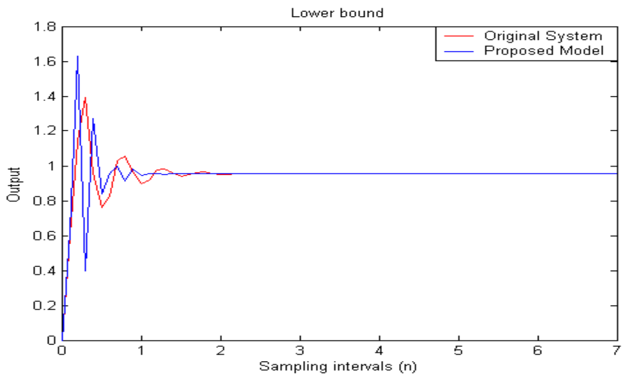
Consider a stable 7th order original discrete time interval system given below:

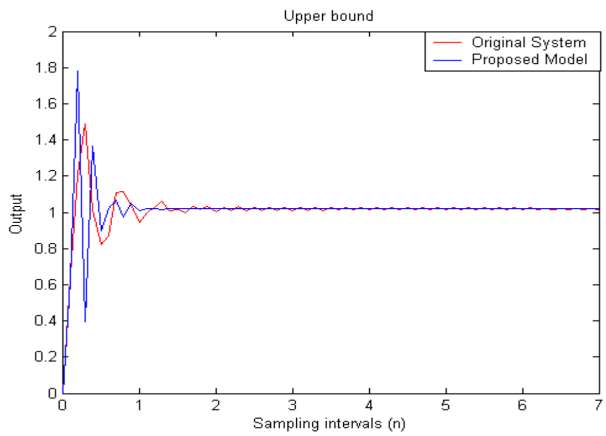
$$G_7(z)=\frac{N_6(z)}{D_7(z)}$$

$$=\frac{[3.25,3.65]z^6+[11.79,13.21]z^5+[19.41,21.74]z^4+[18.48,20.7]z^3+[10.36,11.6]z^2+[3.114,3.488]z+[0.3808,0.4265]}{[5.4,5.65]z^7+[14.77,15.51]z^6+[16.82,17.66]z^5+[13.38,14.05]z^4+[10.58,11.11]z+[6.516,6.842]z^2+[2.182,2.291]z+[0.2856,0.3]}$$

For the above original 7th order interval system, the 2nd order reduced model by the proposed method is obtained as given below:

$$R_2(z)=\frac{[18.52,19.434]z+[48.265,55.381]}{[29.604,31.065]z^2+[29.649,31.13]z+[10.681,11.218]}$$





MIMO Uncertain System

Consider a stable 6th order two input, two output interval system given below in transfer matrix form:

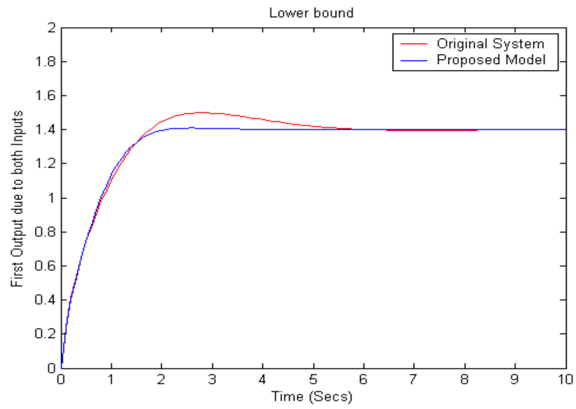
$$\mathbf{G}_6(s)=\cfrac{\begin{bmatrix}[2.0,2.2] & [1.0,1.1]\\ [1.0,1.1] & [1.5,1.6]\end{bmatrix}s^5+\begin{bmatrix}[70,72] & [38,39.5]\\ [30,30.9] & [42,43.5]\end{bmatrix}s^4+}{[1.0,\ 1.1]s^6+[41,\ 42]s^5+[571,\ 576]s^4+}$$

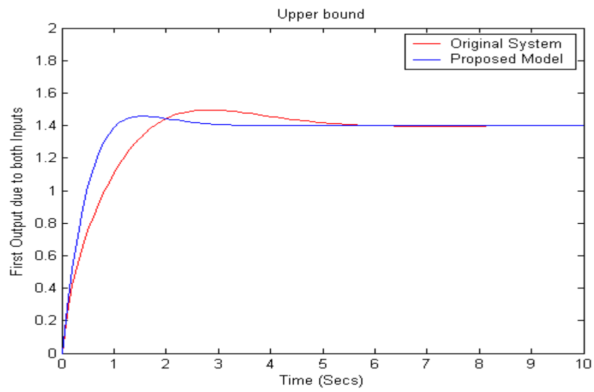
$$\cfrac{+\begin{bmatrix}[762,784] & [459,472]\\ [331,334] & [601,619]\end{bmatrix}s^3+\begin{bmatrix}[3610,3718] & [2182,2248]\\ [1650,1690] & [3660,3769]\end{bmatrix}s^2+}{+[3491,\ 3525]s^3+[10060,\ 10160]s^2+}$$

$$\cfrac{+\begin{bmatrix}[7890,7931] & [4225,4285]\\ [3785,3811] & [9100,9150]\end{bmatrix}s+\begin{bmatrix}[6000,6180] & [2400,2472]\\ [3000,3090] & [6000,6180]\end{bmatrix}}{+[11215,\ 11362]s+[6000,\ 6180]}$$

For the above original high order internal system, the 2nd order reduced model by the proposed method is obtained as given below:

$$\mathbf{R}_2(s)=\frac{\begin{bmatrix}[2.01,2.01] & [1.0,1.0]\\ [1.0,1.0] & [1.48,1.48]\end{bmatrix}s+\begin{bmatrix}[8.8583,9.2147] & [3.5433,3.6859]\\ [4.4292,4.6074] & [8.8583,9.2147]\end{bmatrix}}{[1,1]s^2+[5.5192,5.6471]s+[8.8583,\ 9.2147]}$$





Conclusion

In real application, the order of the system is very high and difficult to analysis, design and control. Model reduction has become a tool in analysis and simulation of high-order variable systems. For these reasons, it is better to represent a high –order system of SISO continuous, discrete and MIMO uncertain systems by a lower –order model without giving the significant features of the original system. The numerator polynomial and denominator polynomial are obtained by using modified δ and γ table respectively. The proposed Affine Arithmetic method as become a better appliance for analysis and simulation of high-order uncertain systems.

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